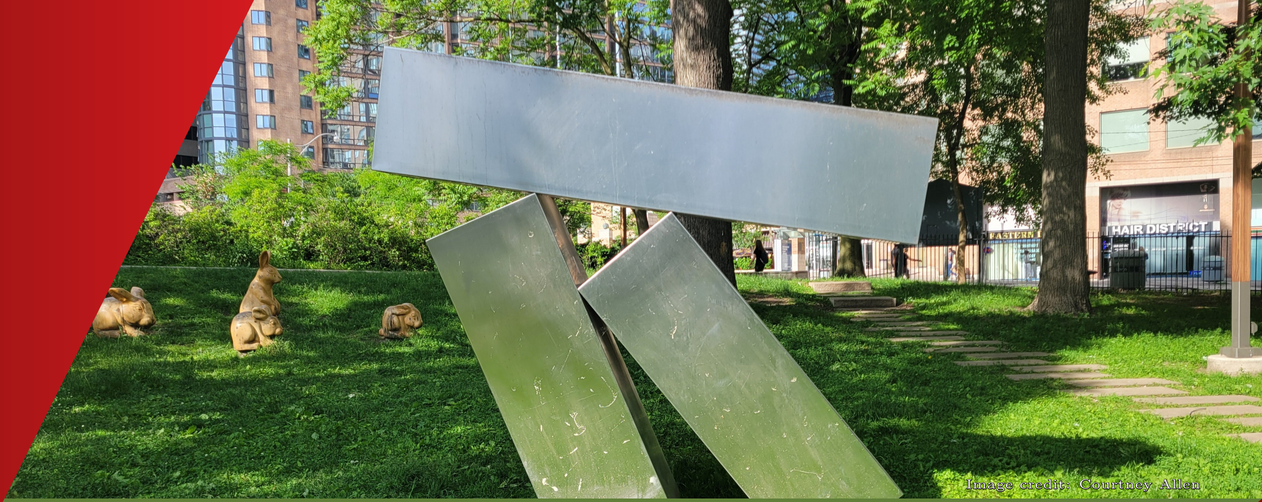


Image credit: Courtney Allen

Notes from the Margin

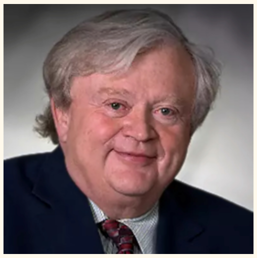
Volume XVIII · 2025

Remembering Robert Woodrow

By: Courtney Allen (University of New Brunswick)

The Canadian Mathematical Society created the Student Committee in 1999 to serve the interests of Canadian mathematics students. That same year, Dr. Robert Woodrow became the Student Committee's faculty advisor. He served on the committee for 26 years, until his death in June of this year. To say that he had an immense impact on our community would be an understatement.

Robert held many roles. As a mathematician, he was well-known for his work on Structural Ramsey Theory. As a professor at the University of Calgary, he was an esteemed educator and mentor, having received the Order of the University of Calgary in 2010.


Robert Woodrow

► 1948-2025 ◄

> *Robert was a beloved colleague who embodied generosity, curiosity, playfulness and selflessness. He took on a range of demanding leadership roles at the university, working tirelessly to create an environment where others could thrive. He was committed to mathematical excellence and dedicated much of his time - even in retirement - to encouraging young mathematicians by supporting mathematics competitions at the provincial and national level. He was also famously unpretentious and informal in his appearance, demonstrating beyond doubt that it's not what's on the outside but what's on the inside that counts.*
>
> – Tony Ware
> (Head of the Department of Mathematics and Statistics at UCalgary)

As a member of the Canadian Mathematical Society, he volunteered on both the Student Committee and Math Competitions Committee, using his free time to serve the needs of future mathematicians in Canada.

> *I am really badly missing Robert for so many reasons, and I am very grateful for his life for those same reasons: he was one of the kindest and most compassionate people I have known, he truly enjoyed mathematics and learning what our students had been working on, he always made time to ensure that students had received all the marks they deserved on a contest, he had great wisdom when it came to dealing with allegations of cheating and preventing this from happening again, he was involved in so many ways with our competition program that I needed four people to replace him this year, and he was a great friend. The only thing I regret is that I did not get a chance to tell him in person how grateful I am for everything he has done for our community and for his friendship. He always thought of others before he would think of himself, and he was truly delighted in the work of our students.*
>
> – Dorette Pronk
> (Chair of the CMS Math Competitions Committee)

In 2018, the CMS recognized him for his contributions by naming him in the CMS Inaugural Class of Fellows.

[continued on page 3]

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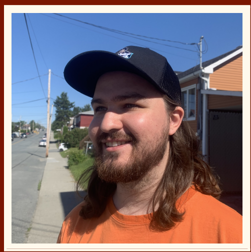
Preamble

By: Jérémy Champagne (University of Waterloo)



Jérémy Champagne
Editor-In-Chief

L'important n'est pas de bien ou mal parler, mais de parler,

– Pierre Falardeau

What is the point of a student journal? The simplest answer is: we are a space for students to publish. There are many reasons why a student would like to be published: to share some material that they find important, to tell the community about what interests them, to celebrate their accomplishments, to improve their writing skills, to gather publishing experience, etc. No matter the reason, articles are what binds the mathematical community together and, in the end, publishing is what we strived towards. *The Margin* has a very high acceptance rate to maximise the number of students that get a chance to publish. It may happen that we receive articles that are not suitable for the journal, but we prefer adjusting them than refusing them, and I hope that it stays that way. So, if there is something that you want people to read: send it our way and we will publish it, one way or another.

This specific issue of *The Margin* starts with a sad reality, which is the passing of Robert Woodrow, who had been the Faculty Representative for the CMS Student Committee since before I even knew how to count. I want to thank Courtney Allen, former Editor of *the Margin*, for writing an article in his honour, which I encourage you to read if you haven't done so already. On the brighter side, this issue is marked by several high quality articles from our contributors, exploring topics such as Galois Theory, Lagrangian Mechanics and Spectral Geometry. We are also continuing our partnership with the *Women in Mathematics* Committee at the University of Waterloo, so make sure to have a look at the articles that they sent us as part of their Directed Reading Program. Bonne lecture!

In This Issue:

[continued from cover]

> *The news of Robert's passing is a profound loss to the Canadian mathematical community, but it will be felt most deeply by those he mentored and encouraged. Robert was a tireless advocate for students, always ensuring their voices were heard and valued within the CMS. His guidance helped shape the work of the Student Committee, and his passion for nurturing the next generation of mathematicians was unmatched. To many, he was not only a mentor but also a source of inspiration and reassurance. Robert's absence leaves a void, but his legacy will live on through the people and programs he so deeply cared about.*
>
> – Termeh Kousha
> (Executive Director of the CMS)

Of course, having served on the CMS Student Committee since its inception, Robert's impact on it was immense. Former Student Committee Chair and founder of Notes from the Margin, Kseniya Garaschuk, wrote an article in CMS Notes about Robert Woodrow shortly after his passing. I've included an excerpt here, but I highly recommend you read the whole piece.

> *In retrospect, what struck me most was the juxtaposition of Robert's undivided attention to Student Committee discussions and his ability to let us work through issues without intervention. First of all, he was fully listening and observing — no phone, computer or a math paper in sight. And secondly, he didn't interject unless prompted or absolutely necessary: he would prevent disasters, but would otherwise let us make our novice mistakes, teaching us the invaluable skill of knowing when to ask for guidance. You felt both supported and capable. He was the perfect safety net that you didn't know was there until it saved you.*
>
> – Kseniya Garaschuk

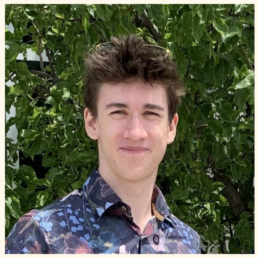
I knew that the 2025 CMS Summer Meeting would mark my last Student Committee meeting, but I never imagined it would be Robert's last meeting as well. It seemed like he'd always be on the committee. Students came and went, but Robert remained, with a wry smile and a subtle sense of humour that put new members at ease. We all knew, intrinsically, that we could count on Robert. When I was organising the student poster session and needed a last-minute replacement judge, Robert was there. But now he isn't.

Robert Woodrow was the rare kind of person who could always be depended upon, and whose significance is often overlooked until it is absent. So all of us Student Committee members, both past and present, want to acknowledge Robert's importance to our community. He will be missed.

Containing the infinite with the Kronecker-Weber theorem

By: Hideki Hill (University of Toronto)

In the previous issue, we discussed foundational field theoretic concepts (extensions, algebraicity, algebraic closures) and understood the meaning of transcendence, especially relating to the number π [4]. In this article, we shall introduce the background necessary to state the Kronecker-Weber theorem.



Hideki Hill

Algebra is generous; she often gives more than is asked of her.

– Jean le Rond d'Alembert

Each of the following definitions are found in [3]: a primitive n-th root of unity is the complex number $\zeta_n := e^{2\pi i/n}$ or one of its powers ζ_n^k with $\gcd(k,n) = 1$. Each primitive root is said to *generate* a finite extension of $\mathbb{Q}$ in $\mathbb{C}$, called a *cyclotomic* field, denoted $\mathbb{Q}(\zeta_n)$ (this notation means we adjoin ζ_n to $\mathbb{Q}$).

Remark. The terms "finite extension" and "finite field" should not be confused. A finite extension is an extension generated by finitely many elements and may contain infinitely many elements, whereas a finite field contains strictly finitely many elements.

The nth roots of unity are frequently described as being the roots of what Busam and Freitag [2] call the "cyclotomic equation", $p(x) = x^n - 1$, not to be confused with the cyclotomic polynomial. Dummit and Foote justify the following equation as a consequence of the Fundamental Theorem of Algebra [3]:

$$x^n - 1 = \prod_{k=0}^{n-1} (x - \zeta_n^k)$$

Notice that the above polynomial is, by construction, a product of unique linear factors with coefficients in $\mathbb{Q}(\zeta_n)$, since $\zeta_n^k \neq \zeta_n^j$ for every $0 \leq j, k < n$ unless $j = k$. After acknowledging that each ζ_n^k for $0 \leq k < n$ corresponds to exactly one vector on the unit circle in the complex plane, one quickly understands this fact intuitively.

We shall now define some terminology related to polynomials that is relevant to our discussion of Galois theory. Each of the following definitions are found in [3]: a polynomial in $\mathbb{F}[x]$ is called *irreducible* over $\mathbb{F}$ if it cannot be factored into a product of polynomials in $\mathbb{F}[x]$. A polynomial in $\mathbb{F}[x]$ is said to *split* over an extension $\mathbb{K}/\mathbb{F}$ if it can be written as a product of irreducible linear factors in $\mathbb{K}[x]$ – further, we say a polynomial is *separable* if each linear factor is unique (i.e. if each root has multiplicity 1).

There are numerous equivalent ways to define what it means for a field extension to be Galois; Artin is known to have formulated many of them in [1]. The following definition of a Galois extension makes use of the theory of polynomials we have developed: an extension $\mathbb{K}/\mathbb{F}$ is called *Galois* if and only if $\mathbb{K}$ is a splitting field for a separable polynomial in $\mathbb{F}[x]$. That is, $\mathbb{K}/\mathbb{F}$ is Galois if and only if a polynomial in $\mathbb{F}[x]$ splits into unique linear factors in $\mathbb{K}[x]$. In the context of cyclotomic extensions, the polynomial $p(x) = x^n - 1$ clearly has coefficients in $\mathbb{Q}$ (the two coefficients are both 1). In its product form, we saw that the polynomial $p(x)$ is a product of *unique* linear factors, and it is clear that each of ζ_n^k for every $0 \leq k < n$ are present in $\mathbb{Q}(\zeta_n)$: we conclude that the nth cyclotomic field $\mathbb{Q}(\zeta_n)$ is Galois over $\mathbb{Q}$.

The following definition of the automorphism group is usually generalized to an arbitrary base field and extension, though we shall only make use of the automorphism group of a cyclotomic field. Under the operation of composition, $\mathrm{Aut}(\mathbb{Q}(\zeta_n)/\mathbb{Q})$ is the group of automorphisms of $\mathbb{Q}(\zeta_n)/\mathbb{Q}$ (isomorphisms between $\mathbb{Q}(\zeta_n)/\mathbb{Q}$ and itself). These automorphisms fix the base field $\mathbb{Q}$ and permute the nth roots of unity. We have seen that the cyclotomic extension $\mathbb{Q}(\zeta_n)/\mathbb{Q}$ is Galois, so we call the automorphism group the *Galois group* and define $\mathrm{Gal}(\mathbb{Q}(\zeta_n)/\mathbb{Q}) := \mathrm{Aut}(\mathbb{Q}(\zeta_n)/\mathbb{Q})$.

Remark. It turns out that every subgroup of the Galois group of a finite extension $\mathbb{K}/\mathbb{F}$ fixes a unique subextension $\mathbb{E}$ (an intermediate extension such that $\mathbb{K} \subset \mathbb{E} \subset \mathbb{F}$), called the *fixed field* of a subgroup. This is known as the fundamental theorem of Galois theory: there is a one-to-one correspondence between subgroups of the Galois group and their fixed fields. See [3] for more on Galois correspondence.

As is the case with any group, the Galois group may be abelian (commutative). In fact, the Galois group of a cyclotomic extension is always abelian; consider that the Galois group of the nth cyclotomic field is isomorphic to the multiplicative group of units modulo n [3]:

$$\mathrm{Gal}(\mathbb{Q}(\zeta_n)/\mathbb{Q}) \cong (\mathbb{Z}/n\mathbb{Z})^\times$$

This fact makes it clear that the Galois group is indeed abelian since it is well-known that the integers commute under multiplication. Computing a composition of automorphisms becomes a matter of the order in which we multiply roots of unity. Again, we are used to this sort of commutative multiplication:

$$(\zeta_n^j)^k = \zeta_n^{jk} = \zeta_n^{kj} = (\zeta_n^k)^j$$

The reason we are interested in identifying abelian Galois groups is the same reason for which we are interested in classifying any group: we improve our understanding of group structures and relations between them. It is convention to describe a Galois extension $\mathbb{K}/\mathbb{F}$ as an "abelian extension" when we really mean that the Galois group $\mathrm{Gal}(\mathbb{K}/\mathbb{F})$ is an abelian group.

We have now developed the sufficient theoretical background to state the Kronecker-Weber theorem.

Theorem (Kronecker-Weber). *Every finite abelian extension $\mathbb{K} \subset \mathbb{C}$ of $\mathbb{Q}$ is contained in a cyclotomic extension $\mathbb{Q}(\zeta_n)$* [5] [7].

The proof of this theorem (too lengthy to present in this article) evaded algebraists and number theorists for decades. The theorem was initially conceptualized and erroneously proven by Kronecker in 1853 [5], and Weber earned his namesake by partially correcting Kronecker's proof in 1886 [7]. Hilbert was the first to complete the proof in its entirety, though his proof used class field theory (an advanced area of mathematics) and is well beyond the scope of this article. Neumann (Olaf, not John von!) corrected and modernized Kronecker's and Weber's original proofs in 1981; this proof is accessible, being mostly contained within the theory discussed in this article [6].

On the surface, the Kronecker-Weber Theorem may appear as another tedious and highly specific result in the vast expanse of abstract algebraic theory, but I wish to convince the reader that this theorem is significant. I perceive its significance most profoundly in its immediate consequence of admitting an explicit largest (maximal) abelian extension in $\mathbb{C}$ over $\mathbb{Q}$: the infinite union of all cyclotomic fields $\mathbb{Q}(\zeta_n)$. To me, this is an excellent display of the ability of mathematics to concretely describe a limitation of the infinite. ◀

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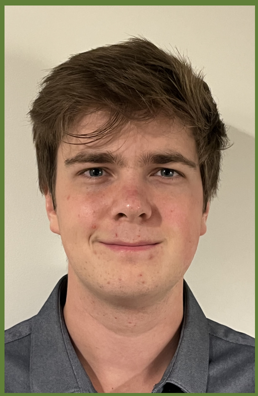
The Calculus of Variations

By: Paul Richter (Thompson Rivers University)

In high school or early university physics you probably learned about Newton's second law,

$$\sum_i \vec{F}_i = \frac{\partial \vec{p}}{\partial t} = m\vec{a}, \qquad (1)$$

which, although not very mathematically (or physically) interesting, accurately describes most of our day-to-day physical phenomena. What you probably didn't learn about was the Lagrangian formulation of classical mechanics, made famous by Joseph-Louis Lagrange in his great *Mécanique Analytique* (1788) [5]. Lagrangian mechanics is a much more powerful and mathematically elegant way of describing the same phenomena. It is foundational to virtually every branch of modern physics in fields as diverse as fluid dynamics and the behaviour of black holes.



Paul Richter

I regard as quite useless the reading of large treatises of pure analysis.

– Joseph-Louis Lagrange

The foundational quantity of Lagrangian Mechanics is the Lagrangian, $\mathcal{L} = T - V$. Here, the kinetic energy T is a function of the velocity $\mathbf{r}'(t)$, the potential energy V is a function of the position $\mathbf{r}(t)$, and the Lagrangian $\mathcal{L}$ is indirectly a function of the velocity and position. The action $S(\mathbf{r}) = \int_{t_0}^{t_1} \mathcal{L}(t, \mathbf{r}, \mathbf{r}')dt$ represents how the Lagrangian (and therefore the kinetic and potential energies) depend on the trajectory $\mathbf{r}(t)$. The theorem underlying Lagrangian mechanics is Hamilton's Principle (1834).

Theorem 1 (Hamilton's Principle)**.** *For initial and final positions $\boldsymbol{r}(t_0)$ and $\boldsymbol{r}(t_1)$, the path of a particle $\boldsymbol{r}(t)$ is an extremal of the action $S(\boldsymbol{r}) = \int_{t_0}^{t_1} \mathcal{L}(t, \boldsymbol{r}, \boldsymbol{r}')dt$ where $\mathcal{L}$ is the Lagrangian [4].*

The action is a **functional**, a mapping from a function to the real numbers, as opposed to regular functions which are mappings from numbers to other numbers. An **extremal** is a function (sometimes called a path) for which a functional has an extrema. Determining the extremals of functionals is the focus of the **calculus of variations**, which we will use to derive a key result in Lagrangian mechanics: the Euler-Lagrange equation, first derived by Euler (1744) but later improved and applied by Lagrange [2].

We'll work with a functional of the form

$$J(y) = \int_{x_0}^{x_1} f(x, y(x), y'(x))dx, \qquad (2)$$

where $J : C^2[x_0, x_1] \to \mathbb{R}$ is a mapping from a function y with continuous second derivatives on $[x_0, x_1]$ to the real numbers. We want to find the local extrema of J in some subset B contained in $C^2[x_0, x_1]$. We impose the restriction that $y(x_0)$ and $y(x_1)$ are independent of our choice of y so that $y(x_0) = y_0$ and $y(x_1) = y_1$. This restriction means that we are simplifying the problem to the **fixed endpoint variational problem**. There are also solutions to the variable endpoint problem, but they are markedly more complex. Physically, this restriction means that we're only considering paths y between some initial and final positions y_0 and y_1.

You'll need to adapt your calculus instincts somewhat when working with the calculus of variations. Rather than thinking of a small change in an independent variable Δx, we need to think of a small change in a function Δy. We can represent this small change as a perturbation of y such as $\hat{y} = y + \epsilon\eta$, where ϵ is a small real number and η is any function that ensures that $\hat{y}$ remains in B. We let H denote the set of all such functions η, that is $H = \{\eta \in C^2[x_0, x_1] : y + \epsilon\eta \in B\}$.

Note that $\eta(x_0) = \eta(x_1) = 0$ because we are working with fixed endpoints, and that $\frac{\partial y}{\partial \epsilon} = \eta$ because y itself is simply $\hat{y}$ evaluated at $\epsilon = 0$. Similarly, $\frac{\partial y\prime}{\partial \epsilon} = \eta\prime$.

Your calculus instincts might tell you to solve $\frac{dJ}{d\epsilon} = 0$ in order to find the extremals. The problem is that we aren't yet sure that this reasoning is valid for functionals; we need to begin with a more rigorous definition of a local maximum of J.

Definition 1. *J is said to have a local maximum in B at y if there is some $\epsilon > 0$ such that $J(\hat{y}) - J(y) \leq 0$ for all $\hat{y}$ such that $||\hat{y} - y|| < \epsilon$ [1].*

A local minimum of $J(y)$ is simply defined as a local maximum of $-J(y)$. In order to find $J(\hat{y}) = J(y + \epsilon\eta)$, we can use the Taylor expansion of f with respect to ϵ to find that, for small ϵ

$$\begin{aligned} f(x, \hat{y}, \hat{y}\prime) &= f(x, y, y\prime) + \epsilon \frac{df}{d\epsilon} + \mathcal{O}(\epsilon^2) \\ &= f(x, y, y\prime) + \epsilon \left(\frac{\partial f}{\partial y}\frac{\partial y}{\partial \epsilon} + \frac{\partial f}{\partial y\prime}\frac{\partial y\prime}{\partial \epsilon} \right) + \mathcal{O}(\epsilon^2) \\ &= f(x, y, y\prime) + \epsilon \left(\frac{\partial f}{\partial y}\eta + \frac{\partial f}{\partial y\prime}\eta\prime \right) + \mathcal{O}(\epsilon^2). \end{aligned} \tag{3}$$

So, we find that

$$\begin{aligned} J(\hat{y}) - J(y) &= \epsilon \int_{x_0}^{x_1} \left(\eta \frac{\partial f}{\partial y} + \eta\prime \frac{\partial f}{\partial y\prime} \right) dx + \mathcal{O}(\epsilon^2) \\ &= \epsilon \delta J(\eta, y) + \mathcal{O}(\epsilon^2), \end{aligned} \tag{4}$$

where δJ is called the first variation of J. We can use integration by parts on the second term of the integrand to find that

$$\begin{aligned} \int_{x_0}^{x_1} \eta\prime \frac{\partial f}{\partial y\prime} dx &= \eta \frac{\partial f}{\partial y\prime}\bigg|_{x_0}^{x_1} - \int_{x_0}^{x_1} \eta \frac{d}{dx}\left(\frac{\partial f}{\partial y\prime}\right) dx \\ &= -\int_{x_0}^{x_1} \eta \frac{d}{dx}\left(\frac{\partial f}{\partial y\prime}\right) dx, \end{aligned} \tag{5}$$

where we've used the fact that $\eta(x_0) = \eta(x_1) = 0$ to evaluate the term outside of the integral as 0. Applying Equation 5 we find that the first variation of J, as seen in Equation 4, can be written as

$$\delta J(\eta, y) = \int_{x_0}^{x_1} \eta \left(\frac{\partial f}{\partial y} - \frac{d}{dx}\left(\frac{\partial f}{\partial y\prime}\right) \right) dx. \tag{6}$$

Recall that if $J(y)$ is a local maximum, the sign of $J(\hat{y}) - J(y)$ must not change for *any* $\hat{y} = y + \epsilon\eta$ such that $||\hat{y} - y|| < \epsilon$. It is clear that $-\eta$ is in H and $\delta J(\eta, y) = -\delta J(-\eta, y)$. From Equation 4, we know that δJ and $J(\hat{y}) - J(y)$ must have the same sign for all η. So, we conclude that $\delta J(\eta, y) = 0$ for all η in H. When applied to the action $S(\mathbf{r})$ in Theorem 1, this requirement is sometimes called the principle of stationary action, and, among other things, forms the foundation for Feynman's path integral formulation of quantum mechanics [3].

You might intuitively see that, because η is an arbitrary function, the other factor in the integrand is necessarily zero. This fact requires the following lemma, which has a fairly simple proof by contradiction that won't be shown here [1].

Lemma 1 (The Fundamental Lemma of the Calculus of Variations)**.** *Suppose that $\int_{x_0}^{x_1} \eta(x)g(x)dx = 0$ for all $\eta \in H$. If $g : [x_0, x_1] \to \mathbb{R}$ is a continuous function then $g = 0$ on the interval $[x_0, x_1]$.*

Combining Lemma 1 and $\delta J(\eta, \epsilon) = 0$, we finally arrive at the Euler-Lagrange equation and its associated theorem [1].

Theorem 2 (The Euler-Lagrange Equation)**.** *For a functional of the form described in Equation 2, if $y \in B$ is an extremal of J, then*

$$\frac{d}{dx}\left(\frac{\partial f}{\partial y\prime}\right) - \frac{\partial f}{\partial y} = 0. \tag{7}$$

You might notice that, if we did take $\frac{\partial J}{\partial \epsilon} = 0$ at the beginning of the derivation, we would arrive at the same result with a bit less work. This is usually the derivation that physics textbooks follow [2]. If we apply Theorem 2 to the action in Theorem 1 and reverse the order by convention, we arrive at the Lagrange equations of motion,

$$\frac{\partial \mathcal{L}}{\partial x_i} - \frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot{x_i}}\right) = 0, \quad i = 1, 2, 3, \tag{8}$$

where the x_is are the three spatial dimensions [2]. The Euler-Lagrange equation is among the most important mathematical results in physics. It is not an exaggeration to say that it is foundational to virtually all of modern physics; oftentimes, writing down the Lagrangian of a system and looking for solutions to the Euler-Lagrange equation is the very first step of investigating new theoretical physics. ◄

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Le monde fascinant de la géométrie spectrale

Par: Alain Didier Noutchegueme (Université de Montréal)

La géométrie spectrale fait partie de ces domaines de mathématiques fondamentales qui trouvent des applications immédiates dans les inventions humaines. Pour l'illustrer, considérons la musique.

Alain Didier Noutchegueme

Les mathématiques sont l'art de donner le même nom des choses différentes

– Henri Poincaré

Dans l'histoire de l'humanité, il y a toujours eu de la musique : Pour séduire, pour les hymnes nationaux, et même pour retenir des informations. C'est pour cela que beaucoup de religions ont ce qu'on appelle des cantiques : lire des écritures en les chantant est un bon moyen de les mémoriser.

Ceux qui jouent de la guitare savent bien que la longueur des cordes et le type de matériaux sont déterminants pour le type de son qui en sort. C'est d'ailleurs essentiellement ce qui fait la différence entre une guitare normale et une guitare basse. La position dans le temps d'une corde de longueur ℓ et fixée à ses deux extrémités est gouvernée par *l'équation de la corde vibrante* ou *équation des ondes* :

$$\frac{1}{a^2}\frac{\partial^2 u}{\partial t^2} - \frac{\partial^2 u}{\partial x^2} = 0, \quad t \in [0, +\infty),\ x \in [0, \ell].$$

Ici, $u(t, x)$ représente la hauteur à la position x de la corde à l'instant t. Si l'on s'intéresse cette fois-ci aux vibrations d'un tambour $\Omega \subset \mathbb{R}^2$, alors cette équation s'étend en dimension d'espace supérieure par la formule analogue :

$$\frac{1}{a^2}\frac{\partial^2 u}{\partial t^2} - \Delta u = 0, \quad t \in [0, +\infty),\ x \in \Omega,$$

où $\Delta := \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$ est le laplacien bi-dimensionnel. Cet opérateur différentiel apparait dans de nombreux phénomènes physiques parcequ'il décrit très bien le monde réel de par ses propriétés de symétrie et d'invariance spatiale. En effet, tout opérateur différentiel linéaire S qui est invariant par translation et par rotation est essentiellement une combinaison linéaire de Laplacien, i.e $S = a_1\Delta + \cdots + a_k\Delta^k$ (voir [2]).

Pour revenir à notre guitare, une fois qu'on la joue, elle émet des sons à une certaine fréquence. Les fréquences (qui se mesurent en Hertz) sont en gros le nombre de vibrations émises par la corde par seconde. Elles sont uniquement déterminées par a et ℓ. Un des enjeux de la géométrie spectrale sera d'étudier jusqu'à quel point, avec les yeux fermés, rien qu'en écoutant de la musique, on peut déduire les caractéristiques de la guitare. Pour y parvenir, la géométrie spectrale est bâtie sur des fondations mathématiques solides. Le but ici ne sera pas de présenter un exposé rigoureux, mais plutôt de motiver le lecteur à chercher à suivre plus tard un vrai cours de géométrie spectrale.

Imaginez que vous ayez une matrice symétrique réelle A, qui est donc diagonalisable. Toute l'information de la matrice A est contenue dans ses valeurs propres et vecteurs propres. D'ailleurs, en analyse de données c'est comme cela qu'on repère les directions qui ont le plus d'information : le premier vecteur propre qu'on appelle premier axe factoriel, indique la direction portant le plus d'informations.

Puisque les matrices ne sont que des opérateurs linéaires agissant sur $\mathbb{R}^n$, une question naturelle serait : *Que se passe-t-il lorsqu'on remplace $\mathbb{R}^n$ par un autre espace muni d'un produit scalaire, mais cette fois-ci de dimension infinie ?* Les premiers espaces auxquels on pense sont :

$$\ell^2 : \text{ Suites } (a_n)_{n=1}^{\infty} \text{ avec } \sum_{n=1}^{\infty} |a_n|^2 < \infty \text{ et}$$

$$L^2(\Omega) : \text{ Fonctions mesurables } f \text{ avec } \int_{\Omega} |f|^2 dx < \infty.$$

Ces espaces sont de dimension infinie. Ceci tombe bien parce qu'on peut définir d'une bonne manière le laplacien comme un opérateur sur $L^2(\Omega)$. Laplacien qui rappelons le, contient toute l'information physique d'un objet.

Du coup, en ''Diagonalisant" le laplacien, on aurait toute l'information physique entre nos mains grâce à une suite de nombres qui tend vers l'infini et les fonctions propres associées. On pourrait définir de manière très réductrice la géométrie spectrale comme *l'art de diagonaliser le laplacien et de tirer l'information géométrique qui y est contenue.*

Voici trois catégories de problèmes auxquels le géomètre spectral s'intéresse au quotidien [1] :

Problème 1. Problèmes inverses : *Quels sont les invariants spectraux ?* i.e : Si je connais toutes les valeurs propres du laplacien, quelle information géométrique cela me donne ? Dans notre monde, cela revient à demander : *Peut-on entendre la forme d'un tambour ?*

Problème 2. *Quelle est la structure des fonctions propres ?* L'étude des zéros des fonctions propres est liée à plusieurs domaines des mathématiques à l'instar de la théorie des nombres.

Problème 3. Optimisation de forme : *Parmis toutes les surfaces d'aire unité, quelle est la géométrie de celle avec la plus grande première valeur propre ?*

La géométrie spectrale est un domaine fascinant, moderne et à fort potentiel applicatif. Cependant, bien que les questions spectrales sont assez vielles, les résultats (en l'occurrence en optimisation de forme) sont peu nombreux. La raison n'est pas juste un manque d'intérêt, mais la difficulté des issues. Ces questions sont faciles à énoncer, mais difficiles à prouver. C'est un domaine qui a besoin de nouvelles idées. Mon souhait est que l'un des lecteurs de cette chronique-ci, révolutionne ce domaine dans l'avenir.

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Women in Math:
Directed Reading Program Articles

For me, the formula for math research has often been unclear. Unlike in other STEM fields, math research doesn't explode from a beaker, squeeze into a pipette, or reveal itself under a microscope. As an undergraduate, I couldn't even picture what generating new math looked like. In class, I learned about classical results from mathematicians who didn't look like me, and on homework, I proved statements I already knew were true. While I now have first-hand research experience as a grad student, other questions have emerged. Once I graduate, how will I choose a new question to work on, and how will I guide my (future) students through dissertations of their own? Luckily, the Directed Reading and Research Program (DRP) at the University of Waterloo is in the business of demystifying math research for undergraduates and graduate students alike.

Organized by Waterloo's Women in Mathematics (WiM) Committee since 2022, the DRP pairs graduate students with undergraduates who are underrepresented in math to work on a project throughout a semester. As DRP mentors, graduate students propose projects in two categories: in the DRP-Reading stream, mentors choose a topic to guide students through, encouraging undergraduate mentees to explore beyond what's covered in their courses. Due to popular demand, WiM launched the DRP-Research stream in 2024. Teams in these projects generate novel results, giving mentees a taste of what math research really looks like and graduate students a chance to step into a supervisory position.

Scan to learn more about the WiM DRP and watch a DRP video.

While the DRP model originated at the University of Chicago and is now an active initiative at over twenty universities, Waterloo's version has a twist. First, by reserving mentee spots for students who identify as minorities in math, our DRP is likely the first to incorporate equity, diversity, and inclusion into the program's makeup. It's also the first faculty-wide DRP, supporting over 30 projects per semester across Waterloo's five math departments. Finally, the delineation between reading and research projects provides opportunities for undergrads at all stages, encouraging them to stick with math. Graduate students also get the unique experience of serving as mentors before they are in a faculty position.

As a DRP-Research mentor, I proved to myself that becoming a supervisor one day may not be so far-fetched. Meanwhile, I saw my mentees grow confident not only in their understanding that math research is the act of constructing new results from the building blocks of what's known, but also in their ability to contribute to this tower of knowledge.

Read on to find accessible introductions to some of our recent DRP reading and research projects.

— Cicely (Cece) Henderson (University of Waterloo)

Solving Partial Differential Equations Using Graph Attention Networks

Applied Math

By: Xena Jiang, Sandy Banh, Alaa Elsayed, Maryam Yalsavar, David C. Del Rey Fernández (University of Waterloo)

Deep learning models have achieved remarkable success in fields such as image and natural language processing. Motivated by these achievements, researchers have recently explored their application to solving Partial Differential Equations (PDEs). However, PDE solution data, generated from simulation software or experiments, is often unstructured, limiting the direct use of conventional deep models designed for structured inputs like images or sequences.

Graph Neural Networks (GNNs) have emerged as a promising framework for handling unstructured data. In the context of PDEs, GNNs represent the computational mesh as a graph and transform nodes/edges embeddings while preserving the graph's structure by learning their relations and patterns. A notable example is the Graph Neural Operator (GNO) [1], which constructs graphs by defining neighborhoods within a ball, and kernel integration is performed through message passing on the resulting graph network. The kernel is learnable and weighs the interaction between nodes. This enables the model to handle varying mesh resolutions effectively. On the other hand, Graph Attention Networks (GATs) [2] introduce an attention mechanism that learns the relative importance of each neighbor. Inspired by attention methods in natural language processing, GATs dynamically weigh contributions from adjacent nodes, allowing the model to focus on the most informative relationships. Despite their success in other domains, GATs have not been widely applied to solving PDEs.

In this work, we first build graphs from PDE solution data using the novel approach proposed in [1], and then train a GAT model on the resulting graphs. We do our experiments on a Darcy Flow benchmark to indicate if incorporating attention mechanisms into graph-based PDE solvers can substantially improve accuracy by enabling the model to learn adaptive representations of node interactions instead of relying on raw neighborhood information alone or not. ◀

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The Interplay Between Tyshkevich Decomposition and Other Graph Operations

Combinatorics and Optimization

By: Helena Devinyak, Cicely Henderson, Hidde Koerts, Boxuan Meng, Molly Wu, and Elaine Zhao (University of Waterloo)

In discrete math, a graph is a set of dots called *vertices* with lines between them called *edges*. From computer science to biology, graphs have many applications because they can model a diverse range of practical problems. Yet in math, we often think of graphs as their own objects worthy of study.

Sometimes we restrict our attention to graphs that share a particular property. For example, a graph G is a *split graph* if it has a KS-partition; that is, the vertex set $V(G)$ can be partitioned into sets K and S where K is a clique (all possible edges appear between the vertices of K) and S is a stable set (no edges appear between the vertices of S).

In our project, we investigated graph operations that take one (or more) graphs as input and systematically output another graph. For instance, just like we can multiply two positive numbers to get a bigger number, we can *Tyshkevich compose* a split graph G with KS-partition $V(G) = K \cup S$ and another graph H to get a bigger graph $G \circ H$ using the following procedure: we keep all of the vertices and edges from G and H and add edges from every vertex in H to every vertex in K. If H happens to be a split graph, then $G \circ H$ is also a split graph, and we can repeat this operation.

Like composite versus prime numbers, a graph is *decomposable* if it's the Tyshkevich composition of at least two other graphs and *indecomposable* otherwise. Inspired by an integer's prime factorization, we say a Tyshkevich decomposition of a graph G is one where each smaller graph in the decomposition is itself indecomposable. In 1980, Regina Tyshkevich proved a powerful theorem [1]: just like every integer has a unique prime factorization, every graph has a unique Tyshkevich decomposition.

There are also many other graph operations: we can make two graphs G and H complete to each other by adding every possible edge between G and H. We can also identify a vertex in G with a vertex in H to glue G and H together, or we can blow up a vertex in G by replacing it with a copy of H.

In our DRP project, we analyzed how these other graph operations affect Tyshkevich decomposition. In particular, given graphs G and H and their Tyshkevich decompositions, we characterized what the Tyshkevich decomposition of $G \cdot H$ is when $\cdot$ is the operation of making graphs complete to one another, gluing along a vertex, or blowing up a vertex. ◄

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Shaperd: Easily Adoptable Real-Time Traffic Shaper for Fully Encrypted Protocols

Computer Science

By: Sina Kamali, Stella Tian, and Sarah Wilson (University of Waterloo)

Fully encrypted protocol-based tools (FEPs) are tools that enable secure communication and are commonly used to circumvent censorship in restrictive regions. FEPs fully encrypt the message payload, removing any and all information about the data, leaving only the *metadata* (any information that is not the data itself) to be observed. Since metadata itself is more than enough to infer information about a communication, FEPs often try to obfuscate and hide the metadata using various techniques, such as making the data seem like a normally allowed protocol (e.g., HTTP).

However, in recent years, censors have been able to block FEPs using an array of attacks based on passive traffic analysis and active probing. In our project, we developed Shaperd, an easily adoptable and real-time traffic shaper designed specifically to aid FEPs to become more resilient to detection. Shaperd operates directly on packet contents in real time, using a novel constraint system to allow its users to generate traffic flows with any desired features. Our preliminary results reveal Shaperd introduces minimal overhead to the underlying system's throughput.

In simple terms, our project showed that it's possible to make censorship-circumvention tools harder to block without slowing them down. Our system, Shaperd, changes the "shape" of internet traffic in real time so that censors can't easily tell which connections are trying to bypass restrictions. We proved that this system works efficiently and adds almost no delay to the network. Our work was recently published in the Free and Open Communications on the Internet workshop as part of the 2025 Privacy Enhancing Tools Symposium.

This project is important because internet censorship affects millions of people around the world, especially in countries where access to information is restricted. Our work helps protect online freedom by making these censorship-resistant tools stronger and more reliable. What we proved matters because it shows that performance and privacy don't have to be a tradeoff. You can have both secure and fast internet access even under heavy censorship.

Our project also paves the way for other researchers working in this field to not have to reinvent the wheel each time they want to create a new tool and be able to easily use our openly accessible code to jump-start their projects. These simple stepping stones are what great projects are built upon. ◄

The Sound of Rough Spaces: Differential Equations on Fractals

Pure Math

By: Roberto Albesiano, Fatma Jadoon, Olivia Larssen (University of Waterloo)

Mathematics studies a diverse array of objects. On one end, there are very smooth shapes, such as spheres, tori, and, more generally, smooth manifolds, where things are well-behaved and nicely differentiable. The smoothness allows for the use of powerful tools from calculus and analysis, and ultimately allows solving partial differential equations.

On the other end of the spectrum, one can find possibly extremely rough shapes, such as fractals.

Although there is no universally agreed upon definition of a fractal, a common property of fractals is *self-similarity*, meaning that a set can be expressed as a union of scaled copies of itself. The Sierpiński Gasket (SG) is a well-known example of a self-similar fractal. It can be constructed by starting with an equilateral triangle and recursively removing the upside-down triangle formed by connecting the midpoints of each side. Another common example of a self-similar set is the unit interval $I = [0, 1]$.

Surprisingly enough, thanks to the self-similar structure, one can import several fundamental tools from classical analysis and differential equations to some fractals. Following Strichartz, with little more than calculus and basic linear algebra, we studied a notion of measure and energy on SG and I, which in the case of I reduce to the classical notions of Lebesgue measure and Dirichlet energy $E(u) = \int_I |u'(x)|^2 dx$.

By interpreting the successive graph approximations of SG and I as resistor networks, we were also able to define a notion of metric called *effective resistance*, which measures the resistance that a battery attached between two points of the fractal would encounter, assuming that all edges have resistance proportional to their length. This metric coincides with the Euclidean metric on I, but is very different from the Euclidean metric on SG, and is much better suited to the analysis on SG.

Combining all these ingredients, we were able to define a Laplacian on SG and I, which in the case of I reduces to the classical second derivative operator. Once one has a notion of Laplacian, one can study several differential equations, such as the heat equation $u_t = \Delta u$ and the wave equation $u_{tt} = \Delta u$. Many surprising facts lie ahead when trying to solve these equations on fractals. How would sound propagate in a Sierpiński Gasket? Would you be able to hear anything at all? ◂

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Elliptic Curves and Fermat's Last Theorem

Pure Math

By: Faisal Al-Faisal, Ellie Hamer, Isabela Souza Cefrin da Silva (University of Waterloo)

The Pythagorean equation $x^2 + y^2 = z^2$ has infinitely many positive integer solutions, such as $(x, y, z) = (3, 4, 5)$ and $(x, y, z) = (5, 12, 13)$. In fact, Euclid already knew how to tabulate all of them. Perhaps this is what inspired Fermat to examine the equation $x^n + y^n = z^n$ for $n \geq 3$. In arguably the most famous *note from the margin*, Fermat claimed that that he possessed a "marvelous proof" that these higher order equations have no solutions in the positive integers. Although Fermat left no indication of how this general proof went, he did at least explain how to deal with the case $n = 4$ (using his method of *infinite descent*).

By examining the prime factorization of n, convince yourself that it suffices to show that there are no solutions when $n = 4$ or $n = p$, an odd prime number. With Fermat taking care of $n = 4$, it fell to Euler to tackle $n = 3$, and then to Dirichlet and Legendre to tackle $n = 5$. All that's left is to handle all remaining primes $p > 5$! But how?

In 1975, Hellegouarch [1] had the idea of taking an alleged solution (a, b, c) to the Fermat equation $x^p + y^p = z^p$ and creating the *elliptic curve* $E_{a,b,c} : y^2 = x(x - a^p)(x + b^p)$. What is an elliptic curve? For the purposes of this note, it's a plane curve defined by an equation of the form $y^2 = f(x)$ where $f(x)$ is a cubic polynomial with no repeated roots. On the face of it, such an object seems to live in the world of algebraic geometry. But elliptic curves have profound connections to various other areas of mathematics. One of the more subtle such connections is to the realm of analysis.

In brief (!), starting from any elliptic curve with rational coefficients, one can build a type of Fourier series (called a *modular form*) with interesting features. The construction is easy to describe—but that it actually works is a consequence of the epoch-making work by Wiles and Taylor [2, 3].

Viewed from this lens, Hellegouarch's elliptic curve $E_{a,b,c}$ is quite unusual: on the one hand, by Taylor–Wiles, one must be able to build a modular form out of $E_{a,b,c}$; on the other hand, Ribet [4] had proved earlier that this was impossible. The only conclusion? The curve $E_{a,b,c}$ *cannot exist*, so neither can the solution (a, b, c) to $x^p + y^p = z^p$. Fermat was right. ◂

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The Financial System in a Warming World

Statistics and Actuarial Science

By: Rhoda Dadzie-Dennis, Alex Palmer, Tanay Kashyap & Kris Zhang (University of Waterloo)

Climate change has shifted from a distant environmental concern to a pressing financial risk. Historically, predictable weather and moderate natural variability allowed markets to value assets with confidence. As these conditions change, new risks are emerging that markets have yet to price in. Our project examines three interconnected areas of climate risks: asset revaluation, changes in human mortality, and rising flood-related liabilities.

The most immediate threat to asset revaluation is transition risk, which arises as the economy moves toward a low-carbon future. To stay below 2°C of warming, most fossil fuel reserves must remain unburned [1], meaning the market value of fossil fuel firms is overstated and a carbon bubble worth trillions looms. A sharp policy or technological shift could rapidly devalue these assets, spreading losses across pension funds and banks, triggering systemic instability.

Climate change is also altering mortality patterns that underpin life insurance and pension systems. Heatwaves have caused a 54% increase in heat-related deaths among the elderly over the past two decades [2], while changing climates are expanding diseases and worsening respiratory illness. Without model updates, insurers and pension funds face growing solvency pressures.

Flood losses now reveal how physical risks destabilize property markets. The 2013 Southern Alberta floods caused over \$5 billion in damage, yet only \$1.7 billion was insured, leaving governments and homeowners to absorb the rest. In Halifax, sea-level rise could endanger \$2 billion in property by 2100, while U.S. homes in flood zones are overvalued by \$34 billion due to underestimated flood risk [3]. As insurers withdraw, falling property values and stressed mortgage-backed securities expose systemic risks similar to those of 2008.

To conclude, climate risks to assets, mortality, and property are closely connected. Floods can erode asset values and worsen health outcomes, while poor policy coordination can deepen market instability. To avert crisis, climate risk must be integrated into all financial decisions, from central bank testing to the management of stranded assets. The threat is no longer distant; it is already reshaping global markets. ◀

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Meta-Analytic Evaluation of Volleyball Metrics

Statistics and Actuarial Science

By: David Awosoga, Yushi Liu, Anna Takegawa, and Allie Dong (University of Waterloo)

Meta-analytics [1] can be used to evaluate the quality of metrics that assess player ability in sports. This helps stakeholders make effective comparisons between players so that they are not overwhelmed by metrics that lack practical significance, technical depth, or appropriate data considerations. Meta-analytics are based on three criteria:

1. **Stability** quantifies how consistent a player's variation in metric m remains across seasons after accounting for random noise, capturing its sensitivity to change.
2. **Discrimination** measures how well a metric differentiates between players as the proportion of average sampling variability to total between-player variance in metric m that reflects true differences in player ability.
3. **Independence** measures how much unique information a metric m adds beyond the set of other metrics $\mathcal{M}$.

In this project we apply meta-analytics to evaluate the quality and underlying properties of a set of both basic [2] and advanced [3–5] volleyball metrics. From this we will be able to identify those that provide the most unique and reliable information for the wider volleyball community, including stakeholders such as coaches, athletes, and fans. Such analysis has not been performed in volleyball analytics literature, and would consequently fill a major gap, particularly with the proliferation of advanced metrics that have been introduced in the past few years. The main objective is to have a greater understanding of the composition of current statistics and identify areas where improvements in independence, stability, and discrimination for new metrics can be made. ◀

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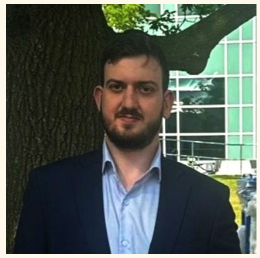
Canadian Undergraduate Mathematics Conference 2025

By: Kareem Alfarra (University of Toronto)

The 2025 edition of the Canadian Undergraduate Mathematics Conference (CUMC) took place between June 23 and June 27, bringing together more than 120 students presenting over 50 talks from a range of universities across Canada. The students got to share their research and knowledge while enjoying a variety of activities, ranging from scientific talks to fun social events.



Kareem Alferra

> Algebra is the offer made by the devil to the mathematician. The devil says: "I will give you this powerful machine, it will answer any question you like. All you need to do is give me your soul: give up geometry and you will have this marvelous machine.
>
> – Michael Atiyah

Starting with academic events, every day included a talk (or two) from one of our six plenary speakers: Jason Bell, Debbie Leung, Florian Girelli and Craig Kaplan from the University of Waterloo; Myrto Mavraki from the University of Toronto; and Grace Yi from the University of Western Ontario. Each of the plenary speakers did a phenomenal job introducing their own subfield and specialty to the conference. In addition, we had an EDI panel on Tuesday to increase awareness among students. We also had a non-academic careers panel on Wednesday, with a variety of panellists from different industries sharing their experience and insights into academia vs industry. We lastly had an academic careers panel on Thursday, including tenured professor Dr. David Jao, recent PhD graduate Dr. Sourabhashis Das, and recently appointed professor Dr. Edward Lee. They shared their experiences and advice on navigating academia.

After long days of back-to-back student talks, there's nothing better to do than to kick back and relax. On the first evening, we had an icebreaker social with food and drinks at the Grad House, the local university pub. Then on Thursday we had two main events. The fun day at Bingemans, the local amusement park, was a great way to unwind and socialize with new friends. Alternatively, we had a library tour of the rare math books collection at the Dana Porter Library for those who preferred to enjoy a quieter activity.

In summary, the 2025 edition of the CUMC catered to a variety of mathematics students from various research backgrounds. It was not only a place to develop new mathematical ideas and learn new topics, but also a place to meet people and form new friendships. Mathematics is never a solitary activity, and it is never more evident than at CUMC! ◀

Acknowledgements

Notes from the Margin **is an official publication of the CMS Student Committee. This edition of *Notes from the Margin* would not be possible without the help of:**

Our Assistant Editor: Teresa Norris.

Our Graphic Designer: Courtney Allen.

Our Advertiser: Maplesoft.

The Women in Mathematics Committee at U. of Waterloo: who provided funding for this issue, and in particular its director, Dr. Yu-Ru Liu, as well as all the students from their Directed Reading Program who submitted articles.

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