

A TALE OF ISOPERIMETRY AND EIGENVALUES

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It has been known since classical antiquity that disks have the largest area among planar figures of prescribed perimeter. Nevertheless, a complete proof was only given around the end of the 19th century! During the 20th century, area and perimeter were replaced by several new analytic and geometric quantities, such as the heat content, torsional rigidity and natural frequencies of vibrations. In this talk, we will survey recent results on isoperimetric bounds for eigenvalues of the Dirichlet-to-Neumann map. This pseudodifferential operator arises naturally in the study of inverse problems that are linked to geophysical and medical imaging. The main question that we will investigate is to find, in various geometric contexts, how large specific eigenvalues can be under isoperimetric constraints. We will see that a vast number of methods can be used, ranging from abstract discretization and expander graphs to homogenization theory, which comes from applied mathematics.