Affine Groups and Continuous Wavelet Transforms

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Abstract: Advances in signal and image processing have been a major theme in scientific and cultural evolution since the invention of the telegraph in the first half of the nineteenth century. Consider a partial list: x-ray images, radio, television, radar, sonar, lidar, movie special effects, MRI, CT-scans, music streaming, Hubble space telescope, etc. All of the advances depend, in one way or another, on the Fourier transform and its remarkable analytic and algebraic properties or close relatives of the Fourier transform. The magic of the Fourier transform comes from the representations of an underlying Abelian group, which might be the real numbers, the unit circle, or a finite cyclic group. About 35 years ago, transforms that arise from the representations of certain non-Abelian groups emerged and led to dramatic improvements in signal and image processing. Collectively, these new tools can be referred to as *wavelet analysis*.

The first part of our presentation will introduce groups of affine transformations and some of the concepts from representation theory that lead to the continuous wavelet transform in one dimension and its two dimensional analogs. The group G_2 of all invertible affine transformations of the plane and some of its properties will also be introduced.

The second part of the talk will present a special irreducible representation of G_2. This representation and its unique properties lead to a new way of transforming and analyzing functions of three variables, which can naturally be thought of as two space variables and one time variable.