Notes from the Margin

Volume III • 2012

Effective sonar through math: How to spot sneaky submarines off your port bow

By Jane Wodlinger (Simon Fraser University)

A *Costas array* is an *n* x *n* grid of dots and blanks, one dot in each row and each column, with the property that no two of the $\binom{n}{2}$ line segments connecting pairs of dots are equal in both length and slope.

They were introduced in the 1960s (in research funded by the US Navy) to model radar and sonar ping patterns; a dot at (i,j) means that frequency f_i is transmitted at time t_j [2]. When the signal is reflected off of a target it is shifted in time and frequency. The distinctness condition on the line segments between dots means that the only translate of the original pattern having high correlation (number of overlapping dots) with the returning echo (even in the presence of noise) is the one whose time and frequency shifts correspond to the target's true position and velocity, respectively (use Figure 1 to convince yourself that this is true).

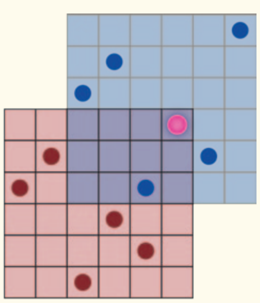


Figure 1: A Costas array of order 6 shown with a translation of itself. For any nontrivial translation, at most one pair of dots overlap (shown in pink).

Early research led to two main algebraic constructions for Costas arrays [2], each using primitive elements of finite fields. For prime p and prime power q, let a be primitive in $\mathbb{F}_p$ and let b and c be primitive in $\mathbb{F}_q$.

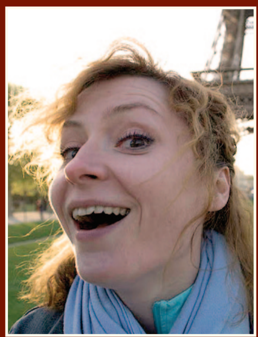
Welch Construction: The array obtained by placing a dot at (i,j) whenever $a^j \equiv i \pmod{p}$ is a Costas array of order p - 1. Further, any cyclic shift of its columns yields another Costas array. The Costas array in Figure 1 is a Welch Costas array generated by the primitive root $a = 3$ in $\mathbb{F}_7$.

Golomb Construction: The array obtained by placing a dot at (i,j) whenever $b^i + c^j \equiv 1 \pmod{q}$ is a Costas array of order q - 2 (see Figure 2). If $b = c$ the array is symmetric.

In addition, there are a number of variations on these two constructions, which involve manipulating a known Costas array (for example, adding or removing a corner dot), if possible, to produce a new Costas array. While the known constructions generate Costas arrays for infinitely many orders (enough to satisfy the engineers), mathematicians (being hard to please) are still searching for answers to some of the most fundamental questions about these combinatorial objects after more than 40 years. Notably, *Is there a Costas array of every order?*

Preamble

By Kseniya Garaschuk (University of Victoria)



Kseniya Garaschuk
Editor

Food-wise, I will try pretty much anything (the other other other white meat, you say?) especially if it comes with a glass of good wine. If at all possible, I'd also get sour cream (aka Belorussian ketchup) on the side.

I shot an elephant wearing my pajamas.

Was I wearing my pajamas or was the elephant? Did I use a rifle or a camera?

Imagine saying this sentence to an audience full of kids — to them an elephant in pink-striped pajamas is a completely plausible (and most definitely a more amusing) scenario. If I am an ivory poacher bragging to my buddies, then I'm surely not talking about my Nikon. I encounter a similar situation every time I face my students in class: will they interpret my words the way I do? If I permute the words, will they still be able to figure out the correct meaning? How can I teach them to have math common sense? How can I teach them to cook up their own solution without giving them a math recipe?

There are two ways of building intelligence: giving the subject a recipe or giving them the opportunity to grow themselves. IBM researchers found the hard way that the former won't get you very far - without being able to *learn* from its 15-terabyte database of facts, the first version of their Watson was able to answer only about 10% of Jeopardy questions correctly. However, facts and rules also have their place: after all, 6 million of them taught Watson to have what we call common sense.

In this issue of the Margin, we focus on teaching and educational aspects of our academic lives. The articles presented here include discussions on two methods of learning and teaching at a university level, in grade 3 and in some unreal situations. Articles about Canadian Undergraduate Mathematics Conference, International Math Olympiad and the research pieces give a preview of different opportunities available in the mathematical world if you devise your own recipes, while the Distractions Page continues distracting.

Consider this magazine to be your cookbook and submit your personal favourite recipes (whether teaching, education or research-related) to student-editor@cms.math.ca.

Happy cooking!

*** I credit Groucho Marx and Nova's "Smartest Machine on Earth" for my elephant and Watson inspirations.**

Jane Wodlinger

I love to bake but I rarely use a recipe, although I once co-wrote a cookbook called "Recipes for Disaster". My other distractions from math include knitting, gardening and (before you mistake me for an old lady) taekwondo.

The answer to this question for order 32 has been called the "Holy Grail" of Costas array research, as this order has been, since 1984, the smallest for which no construction is known [2]. Seeking insight into the existence pattern (and inching towards an exhaustive search for order 32), researchers have enumerated all Costas arrays up to order 29[1]. Of these, roughly 90% are sporadic [1], meaning they don't arise from any of the known construction methods. An exhaustive search for order 32 was recently declared to be within reach, requiring an estimated 45000 years of CPU time (using current algorithms and equipment) [1]. A negative result would settle the 40 year-old existence question once and for all, while the discovery of new sporadic examples may reveal structural constraints or lead to new construction methods. Place your bets now!

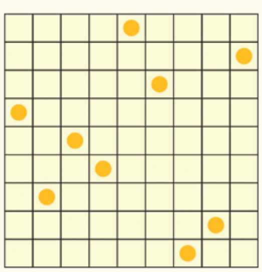


Figure 2: A Golomb Costas array generated using $b = 2$, $c = 7$ over $\mathbb{F}_{11}$

References

[1] K. Drakakis, F. Iorio and S. Rickard. The enumeration of Costas arrays of order 28 and its consequences. Adv. Math. Commun., 5(1):69–86, 2011.

[2] S. Golomb and H. Taylor. Constructions and properties of Costas arrays. Proc. IEEE, 72(9):1143–1163, 1984.

[1] See www.costasarrays.org for current enumeration results and a database of papers.

Using real-world examples with "unreal" students

By Arun Moorthy (University of Guelph)

Upon convocation, we are often recipients of two things: (1) a warm congratulations on our academic accomplishments and (2) a cautious warning that we are now entering the real world.

Apparently, it is quite common for society to differentiate between the working world and the academic as real and unreal. This leads to an important question for educators: How do you use real-life examples to explain theory to a student who has never even been exposed to the real world?

We can give thorough examples about the use of differential equations in process control loops, or Taylor-Series expansion for computer-based solutions; but really, these examples sometimes do more damage than good, reinforcing students' initial intimidations, and further propagating the perception that mathematics is an abstract matter exclusive to the rocket scientists. As a way to help my students, I define unreal-world examples as those that don't require a formal work experience or interest in pursuing a specific career or interest in, well, anything: a requirement perfect for incoming frosh. One quick example is using the baking of cookie dough into cookies to explain how a function (baking) transforms x (cookie dough) into y (cookies). I realize that this is not the most sophisticated description, and definitely not the most thorough description of a function, but thus far I have found it to be quite effective in making calculus seem more accessible. After all, first-year calculus houses students the furthest away from the real world than any other mathematics course offered; hence, using ideas closer to the unreal lifestyle of an 18-year old living away from home for the first time is a no-brainer.

Luckily for us, mathematics happens to be everywhere, and so finding examples of unreal applications isn't all that hard. Actually, I think they might even have a unit for that: *i*?

Arun Moorthy

My favourite foods are often 'street meats', whether it be a hot dog while in Toronto or a lamb kabob in Beijing.

The 2012 Canadian Undergraduate Math Conference will be held from July 11-15 in Kelowna, BC. The event will be co-hosted at UBC's Okanagan campus and the Okanagan College. Those attending will have the opportunity to see what the Okanagan valley has to offer, from beaches and parks, to orchards and vineyards. Highlights of the conference will consist of one day of talks held at the College to showcase the role colleges play in undergraduate education; the conference will also feature the Women in Math and Science Dinner. Confirmed speakers for the conference are Dr. Heinz Bauschke (UBC Okanagan), Dr. Tim Swartz (Simon Fraser University), Dr. Jennifer Hyndman (University of Northern British Columbia), Dr. Gerda de Vries (University of Alberta), and Dr. Catherine Beauchemin (Ryerson University). We invite all undergraduate and graduate students to come and give a talk on any mathematical related topic that interests them. The organizers of the 2012 CUMC look forward to seeing you there! Check us out at cumc.math.ca.

Le Congrès Canadien des Étudiants en Mathématiques 2012 aura lieu du 11 au 15 juillet à Kelowna en Colombie-Britannique. L'événement se déroulera conjointement au campus de l'Okanagan de l'Université de la Colombie-Britannique ainsi qu'au Collège de l'Okanagan. Les participants auront l'opportunité de voir ce que la vallée de l'Okanagan a de mieux à offrir, comme ses plages, parcs, vergers et vignobles. Les points forts de la conférence seront la journée de présentations au collège qui mettront l'emphase sur le rôle que les collèges peuvent avoir face aux études de premier cycle, ainsi que Women in Math and Science Dinner. Les conférenciers qui ont confirmé leur participation jusqu'à présent sont: Dr. Tim Swartz (Université Simon Fraser), Dr. Jennifer Hyndman (Université du Nord de la Colombie-Britannique), Dr. Gerda de Vries (Université de l'Alberta), et Dr. Catherine Beauchemin (Université Ryerson). Nous invitons tous les étudiants de tous les cycles à venir et à présenter sur n'importe quel sujet relié aux mathématiques qui les intéressent. Les organisateurs de la conférence espèrent vous voir en grand nombre! Consultez notre site web: cumc.math.ca.

Infinity in my hands:
How to get kids to explore infinity

By Ricardo Scucuglia Rodrigues da Silva and George Gadanidis (University of Western Ontario)

Ricardo Scucuglia

I am the former guitarist of a Brazilian band named Sancast. Nowadays, I enjoy playing live math concerts at schools in Ontario as an invited participant of the band Joy of X.

(see joyofx.ca)

"To see a World in a Grain of Sand, And a Heaven in a Wild Flower, Hold Infinity in the palm of your hand, And Eternity in an hour." — William Blake, *Auguries of Innocence.*

How do you explore series $\sum(1/2)^n$ at different school levels? In high-school, you can use simple algebra to show the students that the sum converges to 1. In university, you can use first-year calculus to show that this series converges absolutely [1]. But how does one go about explaining this to a much younger audience?

In activities we have conducted in schools, Grade 3 students are given a set of five identical square pieces of paper with square grid on them. On the first square, they shade in a representation of one half. On the second square they shade in one quarter, then one eighth, one sixteenth and so on. We ask them to imagine doing this with more and more squares, on and on, forever. How big an area would be covered if we combined all the shaded parts? Most students suggest that the shaded area would have to be quite large, maybe even infinite in size, since we keep adding to it without stop. Usually at least one student notices that if we use scissors to cut out the shaded fractions, they would all fit inside one of the squares. Taking up this idea, students start with one new square and shade in one half, then one half of the unshaded part, and one half of the unshaded part, and so on. We explore different ways this might be done, leading to a variety of colourful representations. So when we then write on the board "1/2 + 1/4 + 1/8 + 1/16 + ... = ?", although there is some uncertainty at first, students quite quickly convince one another that the answer is either 1 or something very close to 1, since the shaded representations of these fractions fit in "one whole". Since, the fractions never escape the square, the sum cannot be more than 1.

"Infinity in my hand." Rendered by Ann Langeman, designed by George Gadanidis

In this example, students are engaged in the exploration of a visual proof. [2] posits "a number of mathematicians and logicians are now investigating the use of visual representations, and in particular their potential contribution to mathematical proofs". No matter the age of the students, the discovery and exploration of mathematics can occur in any classroom in many different ways. For more ideas on exploring infinity through performance arts, please visit researchideas.ca.

References

[1] Ávila, G. (2009). Análise Matemática para Licenciatura. [Mathematical Analysis for Undergraduates]. Sao Paulo, Brazil: Editora Edgard Bluncher

[2] Hanna, G. (2000). Proof, explanation and exploration: An overview. Educational Studies in Mathematics, Special issue on "Proof in Dynamic Geometry Environments", 44 (1-2), 5-23.

Studc is in the process of re-launching its web-site. The website contains information on all of Studc's projects such as Notes from the Margin, workshops and other events at the semi-annual CMS Meetings, the Canadian Undergraduate Mathematics Conference as well as information on funding support for local student events. Let us know your comments at http://studc.math.ca.

Studc est en train de préparer le lancement de son nouveau site web. Le site contient des informations sur tous les projets menés par Studc tels que la revue "Notes from the Margin", les ateliers et les événements tenus aux réunions semi-annuelles de la SMC et le Congrès Canadien des Étudiants en Mathématiques. Vous y trouverez également de l'information sur le financement offert par le comité pour des événéments étudiants. Faites-nous parvenir vos commentaires à http://studc.math.ca.

Understanding: Relational versus Instrumental

By Amber Church (University of Guelph)

In his seminal article on teaching and learning in mathematics, Richard R. Skemp differentiated two key types of understanding. [1]

The first of these, relational understanding of a concept, is seeing the true nature of the concept. With relational understanding, a student can break down the concept, analyze it and even generalize it to apply the knowledge in new and diverse ways. Students possessing a relational understanding view math as a flow chart and can see how concepts and ideas interweave. The other type of understanding is instrumental understanding, which occurs when a concept is explained as a step-by-step process. There is no asking or explaining "why"; there is just a rule to apply. Students possessing instrumental understanding therefore create for themselves a compartmental outlook toward mathematics, where each idea is its own entity and the connections between concepts are not easily seen.

A problem that often arises in a classroom is miscommunication between a teacher's style of instruction and the students' expected style of learning creating a divide in the learning process. The most common situation occurs when a teacher strives for students to achieve a relational understanding while the students are satisfied with instrumental. At a university level, instructors are often faced with students who "just want to know how to do the question" and this attitude toward mathematics could lay in the way they have been previously taught. Primary teachers build a student's mathematical background and shape the remainder of that child's math career. Unfortunately, teaching toward an instrumental understanding occurs often in schools and, while there are many explanations for it, this approach has serious consequences at a later stage of education. Working with students with different backgrounds, we can strive for a student to achieve relational understanding. However, we need to adapt to the situation and see if instrumental understanding is enough to allow the student to reach their individual goal. Furthermore, we should not be frustrated when all a student wants is a mathematical recipe; after all, it is ultimately their decision.

Skemp's ideas continue being very relevant in mathematical education today: for instance, Ontario educators refer to conceptual and procedural understandings [2]. At the very least, it is critically important that we, as pedagogs, relationally understand the material we are presenting to not only be able to explain how to get the answer, but to "help children see, hear and feel mathematics" [2, p.76].

Amber Church

Growing up in an Eastern European household meant eating all parts of an animal. I learned early to question what was on my plate. Being from Northwestern Ontario also meant eating a lot of deer and moose meat.

References

[1] Richard R. Skemp, "Relational Understanding and Instrumental Understanding", Mathematics Teaching, 77, 20–26, (1976).

[2] "Education for All", Ontario Report on Literacy and Numeracy, http://www.edu.gov.on.ca/eng/document/reports/speced/panel/speced.pdf

What do trace vectors have to do with private quantum channels?

By Sarah Plosker (University of Guelph)

Sarah Plosker

I love to eat - I've tried everything from ostrich steaks to octopus sushi. I once bought a 3 kg wheel of beer cheese: it was $120 and worth every penny!

© digitalart

Private quantum channels are at the heart of quantum cryptography.

The oft described situation is as follows: Alice wishes to send a message to Bob without an eavesdropper, Eve, being able to learn the message. Alice and Bob share a key, i, and Alice applies invertible operators corresponding to i to her message (that is, Alice sends her message through a special channel $\mathcal{E}$). Bob receives the output message, and, knowing i, can undo Alice's operations to recover the original message. This is secure against Eve, who does not know i, and always sees the same output regardless of Alice's input message.

In the literature, examples of private quantum channels have been limited to channels formed using tensor products of Pauli matrices (defined below). Using the machinery of *trace vectors* and *conditional expectations* (see [1] for a theorem regarding the latter), we can give new examples. We will be working in finite dimensions throughout this article. We will denote by $\mathcal{H}_n$ an n-dimensional Hilbert space, $\mathbb{M}_n$ the algebra of n x n complex matrices, and $\mathbb{1}$ the identity element.

Dirac (Bra-ket) notation (1930): A vector is called a "ket" and written as $|\psi\rangle$. Every ket $|\psi\rangle$ has a dual "bra", written as $\langle\psi|$. If $|\psi\rangle = [c_0\ c_1\ c_2\ \ldots\ c_n]^T$ (a column vector), then $\langle\psi| = [\overline{c_0}\ \overline{c_1}\ \overline{c_2}\ \ldots\ \overline{c_n}]$ (where $\bar{c}$ represents complex conjugation of the scalar c). The inner product is denoted by a bracket: $\langle\phi|\psi\rangle$.

Pure states (rank one projections) are represented by $|\phi\rangle\langle\phi|$, where $|\phi\rangle$ is a unit vector (that is, a vector satisfying $\langle\phi|\phi\rangle = 1$). General quantum states are represented by density operators ρ (nonnegative operators with trace equal to 1).

Examples:

- The maximally mixed state $\frac{1}{n}\mathbb{1} \in \mathbb{M}_n$.
- A maximally entangled state has corresponding unit vector $|\varphi_e\rangle = \frac{1}{\sqrt{d}}\sum_{i=1}^{d}|e_i\rangle\otimes|f_i\rangle \in \mathcal{H}_m \otimes \mathcal{H}_n$, where $\{|e_i\rangle\}$ and $\{|f_i\rangle\}$ form orthonormal sets in $\mathcal{H}_m$ and $\mathcal{H}_n$ respectively, and $d = \min\{m,n\}$.

Definition: We say that a map $\Phi : \mathbb{M}_m \to \mathbb{M}_n$ is *completely positive* (CP) if the induced map $\Phi_d : \mathbb{M}_d \otimes \mathbb{M}_m \to \mathbb{M}_d \otimes \mathbb{M}_n$ defined by $\Phi_d = \mathbb{1}_d \otimes \Phi$ is positive for all d. A (*quantum*) *channel* $\mathcal{E}$ is a linear, CP, trace preserving map. Note that channels send density matrices to density matrices.

Examples:

- A channel $\mathcal{E}$ is called a *random unitary channel* if it admits a decomposition

$$\mathcal{E}(\rho) = \sum_i p_i U_i \rho U_i^\dagger \qquad \forall\rho,$$

where p_i form a probability distribution, U_i are unitary operators, and $U_i^\dagger$ is the adjoint of U_i.

- A map $\mathcal{E} : \mathbb{M}_n \to \mathbb{M}_n$ is called a *depolarizing channel* if, for some $0 < p \leq 1$,

$$\mathcal{E}(\rho) = \frac{p}{n}\mathbb{1} + (1-p)\rho \qquad \forall\rho.$$

  We obtain the completely depolarizing channel when $p = 1$.

Based on the Alice & Bob paradigm discussed above, we define a private quantum channel as follows.

The 2012 CMS Summer Meeting will take place June 2-4, 2012, hosted by University of Regina. Studc is planning a number of exciting student events to be held during the meeting, including a poster session, a panel discussion, a writing workshop and a social. Check the meeting website at http://cms.math.ca/Events/summer12/ for more information.

La Réunion d'hiver de 2012 de la SMC aura lieu du 2 au 4 juin 2012 à l'Université de Regina. Studc prévoit un certain nombre de manifestations excitant pour les étudiants qui se tiendra lors de la réunion, notamment une séance d'affiches, un panel de discussion, un atelier d'écriture et un événement social. Vérifiez le site Web de la Réunion à http://cms.math.ca/Reunions/summer12/.f pour plus d'informations.

Definition: Let $\mathcal{S} \subseteq \mathcal{H}_m$ be a set of unit vectors and let $\mathcal{E} : \mathbb{M}_m \to \mathbb{M}_n$ be a channel. Let ρ_0 be a density operator in $\mathbb{M}_n$. Then $[\mathcal{S}, \mathcal{E}, \rho_0]$ is called a *private quantum channel* (PQC) if for any unit vector $|\phi\rangle \in \mathcal{S}$, we have $\mathcal{E}(|\phi\rangle\langle\phi|) = \rho_0$.

Definition: Let $\mathcal{A}$ be $*$-subalgebra of $\mathbb{M}_n$. A vector $|v\rangle$ is a trace vector of $\mathcal{A}$ if

$$\langle v|a|v\rangle = \frac{1}{n}\mathrm{Tr}(a) \quad \forall a \in \mathcal{A}.$$

By letting $a = \mathbb{1}$, we see that trace vectors are necessarily unit vectors.

Examples:

- From the definition, it is clear that the full matrix algebra $\mathbb{M}_n$ has no trace vectors.
- A maximally entangled state $|\varphi_e\rangle \in \mathcal{H}_m \otimes \mathcal{H}_n$. If $m \geq n$, then one can check via direct calculation that $|\varphi_e\rangle$ is a trace vector for the algebra $\mathbb{1}_m \otimes \mathbb{M}_n$. And if $m = n$ an analogous calculation works for $\mathbb{M}_m \otimes \mathbb{1}_n$.

The following three matrices are known as the *Pauli matrices:*

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \ \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \ \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}.$$

Note that the set $\{\mathbb{1}, \sigma_x, \sigma_y, \sigma_z\}$ forms a basis for the real vector space of Hermitian matrices in $\mathbb{M}_2$. Any unit vector $|\psi\rangle \in \mathbb{C}^2$ can be identified with a point $\vec{r}$ (called a *Bloch vector*) on the unit sphere in three dimensional space known as the *Bloch sphere*. Moreover, we can associate to any density operator $\rho \in \mathbb{M}_2$ a Bloch vector $\vec{r}$ via

$$\rho = \frac{\mathbb{1} + \vec{r} \cdot \vec{\sigma}}{2}, \quad \text{where } \vec{\sigma} = (\sigma_x, \sigma_y, \sigma_z)^T.$$

See [2] for details.

Every unital qubit channel $\mathcal{E}$ can be represented as

$$\mathcal{E}\left(\frac{1}{2}\left[\mathbb{1} + \vec{r} \cdot \vec{\sigma}\right]\right) = \frac{1}{2}\left[\mathbb{1} + (T\vec{r}) \cdot \vec{\sigma}\right],$$

where T is a 3 x 3 real matrix that represents a deformation of the Bloch sphere. We are of course interested in cases where $\mathcal{S}$ is nonempty (it would be silly of us to consider a private quantum channel that is private for all vectors in the null set!). We consider the cases in which T has non-trivial nullspace; that is, the subspace of vectors $\vec{r}$ such that $T\vec{r} = 0$ is one, two, or three-dimensional.

Theorem: [1] *Let $\mathcal{E} : \mathbb{M}_2 \to \mathbb{M}_2$ be a unital qubit channel. Then there are three possibilities for a private quantum channel $[\mathcal{S}, \mathcal{E}, \frac{1}{2}\mathbb{1}]$ with $\mathcal{S}$ nonempty:*

1. *If the nullspace of T is 1-dimensional, then $\mathcal{S}$ consists of a pair of orthonormal unit vectors.*
2. *If the nullspace of T is 2-dimensional, then the set $\mathcal{S}$ is the set of all trace vectors of the subalgebra of 2 x 2 diagonal matrices up to unitary equivalence.*
3. *If the nullspace of T is 3-dimensional, then $\mathcal{E}$ is the completely depolarizing channel and $\mathcal{S}$ is the set of all unit vectors. In other words, $\mathcal{S}$ is the set of all trace vectors of $\mathbb{C} \cdot \mathbb{1}_2$.*

Figures 1, 2 and 3 represent the three cases of the theorem.

What's so great about this theorem?

- It links trace vectors, a concept used in matrix theory and first introduced by Murray and von Neumann in 1937 (under the name "u. d. r."), with private quantum channels, which were first studied in 2000 by Ambainis, Mosca, Tapp, and de Wolf, and by Boykin and Roychowdhury;
- It allows us to visualize the set $\mathcal{S}$; that is, we can see exactly which states Alice can send to Bob privately;
- It generalizes quite nicely to arbitrary (finite) dimensions;
- It's a result from my first refereed paper :)

Please see [1] for details, generalizations, and proofs.

References

[1] A. Church, D. W. Kribs, R. Pereira, and S. Plosker. Private Quantum Channels, Conditional Expectations, and Trace Vectors. Quantum Information & Computation (QIC), 11 (2011), no. 9 & 10, 774 - 783.

[2] M. Nielsen and I. Chuang. Quantum Computation and Quantum Information. Cambridge Univ. Press, 2000.

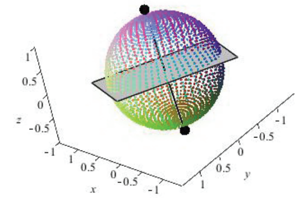


Figure 1: Case (1): The nullspace is a line passing through the origin of the Bloch sphere. The line hits the surface of the sphere at two antipodal points; these points make up S.

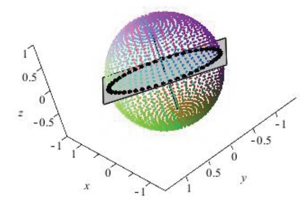


Figure 2: Case (2): The nullspace is a plane passing through the origin of the Bloch sphere. This plane meets the surface of the sphere in a great circle. The pure states corresponding to the points on this circle are precisely the private states of the channel.

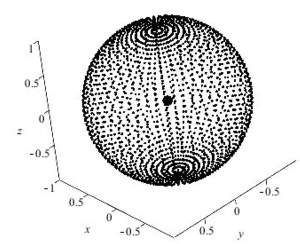


Figure 3: Case (3): All points on the surface of the sphere are mapped to the center. The deformation map T is the zero operator.

Interview with Canadian Math Olympians

By Kseniya Garaschuk (University of Victoria)

The International Mathematical Olympiad (IMO) is the most prestigious annual high school mathematics competition in the world. The IMO began in 1959 in Romania with 7 participating countries. In 2011, the 52^{nd} IMO hosted in Amsterdam welcomed 564 students from 101 countries.

James Rickards

I am an avid glider pilot, earning my license at 16, the youngest age possible. I am out flying most weekends from late spring to mid autumn.

Matthew Brennan

My favourite meal is a burger, with mayo, Swiss cheese and onions, coupled with fries, poutine and a drink of coca-cola. If poutine is not available, then I am fine with the fries alone.

All of the six members of the Canadian team received medals: 1 gold, 2 silver, and 3 bronze. Overall, Canada ranked 17^{th} as a country. We interviewed team members James Rickards and Matthew Brennan to recount their experiences.

How did you prepare for IMO while in Canada?

MB: Before we went to the IMO, we spent two weeks at Banff International Research Station. Our trainers prepared problem sets and six practice Olympiads to simulate the conditions of the IMO.

JR: It was intense. Even after supper we had problem solving sessions. In all, we were probably doing math for over 8 hours every day.

What is a typical day at IMO like?

MB: We spent nine days in total at the IMO. The third and the fourth days were the two days of the contest. On other days, we went on several excursions. Usually we arrived with enough time left in the day to play board games and join in other activities.

Did any teams or people stand out?

JR: There were several individual performances of note this year. Lisa Sauermann of Germany became the most decorated IMOer ever, winning her 4^{th} gold medal with a perfect score. Also of note was the age of many top contestants: Raul Sarmiento from Peru was 6^{th} overall and he is only 13. In fact, this is his 3^{rd} medal! 14-year-olds near the top include Lin Chen from China in 3^{rd} place, David Yang from USA in 4^{th}, and our own Alex Song in 25^{th}.

MB: The Chinese and US teams did the best in the competition, both having all six members of their teams return with gold medals. The British team was always very well dressed - I would vote for them as the most congenial team.

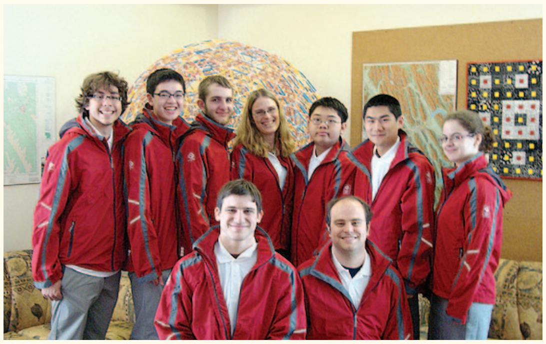
The next IMO will be held in Mar del Plata, Argentina, **July 4-16, 2012.**

Did you get to explore the Netherlands?

MB: The excursions we went on included bowling, sailing, visiting a couple of smaller Dutch towns. We also walked through Amsterdam and saw the city through the canals. The excursion to The Hague was my favourite: I really enjoyed visiting the Escher museum.

What did you notice that was different from North America?

JR: Amsterdam was a lot like any large city in Canada, but with many small differences. In most places bikes had their own path, and many of us continually forgot this. Bicycles are very important there: the laws dictate that bicycles are rarely at fault in collisions. This makes motorists very careful around them - a nice change from many places in Canada.



How did you enjoy the overall experience?

MB: The most memorable parts of the IMO were the contest itself and spending time training with the Canadian team. I remember being very worried for the exam, so solving the first problem was a huge relief.

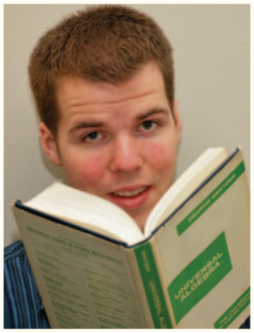
JR: Those three weeks were some of the best of our lives. Thanks goes out to our trainers who helped and supported us the whole way, our guide who did a great job in showing us around, and the many sponsors including the Canadian Mathematical Society and the Samuel Beatty Fund. Without them this would not have been possible. The IMO was a great experience for all.

The Student Committee is inviting undergraduate math students to apply for Canadian Undergraduate Mathematics Conference (CUMC) Award for Excellence. This award is valued at $500 and is given to an outstanding student for the purpose of participating in CUMC. All applications will be judged on university and mathematical community involvement, academic excellence and research potential. The application deadline is March 31st. For application information, please go to http://studc.math.ca.

Le Comité Étudiant invite les étudiants en mathématiques de premier cycle à déposer une demande pour la "Award for Excellence" du Congrès Canadien des Étudiants en Mathématiques (CCÉM). La bourse d'une valeur de 500$ est remise à un étudiant exceptionnel afin de lui permettre de participer au CCÉM. Toutes les demandes seront juger par rapport à l'implication universitaire et dans la communauté mathématique, l'excellence académique et les aptitudes à la recherche. La date limite pour soumettre une demande est le 31 mars. Pour plus d'informations sur la demande, veuillez consulter le http://studc.math.ca.

CUMC 2011: A First Timer's Experience

By David Petersen (University of Northern British Columbia)



David Petersen

Lessons learned from late nights of programming:

1) Pizza Pops cooked in one's computer aren't bad.

2) Cheese is awfully hard to remove from one's computer!

How often do we get the opportunity to informally talk about math? I mean, we all know the expected outcome of an attempted conversation with most people: their eyes glaze over and the encounter is done.

This leaves us, for the most part, with only fellow math people. But, usually, there just aren't enough of us in any one place: my own department only has 2 other people in my year! This problem was the impetus for my quest to attend my first math conference.

Being pretty ignorant about the whole process of conference-going, I decided to talk to someone knowledgeable in such things. Luckily, such people are pretty abundant at a university! The easy part was picking a conference (Canadian Undergraduate Mathematics Conference seems reasonable enough for a Canadian undergraduate mathematics student). The lesson on how to shake the university money tree wasn't too bad; the strong encouragement to give a presentation was terrifying.

Thankfully, I accosted some graduate students who walked me through a procedure to build a presentation:

1. Pick a topic and prepare a draft presentation;
2. Swear at Beamer;
3. Repeat step 2 until insanity sets in.

With my presentation finalized on the plane, I was now ready to go. The conference itself was a bit of an overwhelming blur (in a great way). Here's a rough day-by-day outline:

Day 1: 12 hours of travel, an opening presentation (I've never seen a math lecture with more than 12 students -- wow!), an opening banquet. So many interesting people speaking math. Very strange.

Day 2: The first presentations were both interesting and terrifying. The content was very neat and they were very, very well done. How could I stand up to that? Thankfully, there were also some rougher presentations, which was reassuring in how supportive the audience was. I also discovered that the informal discussions between talks is reason enough to attend a conference! *

*Editor's note: Maybe the only reason...

Day 3: Much like Day 2, but with less fear and more excitement.

Day 4: Presentation day: I learned that before presenting in front a large, relatively unknown audience my knees wobble and my fingers jitter. Luckily, I was on autopilot once I started the presentation. I can't really remember what went on, but the others tell me it went well. The rest of the day was a bit of a haze, but the closing banquet was great. Another chance to talk informally!

Day 5: The end of the conference, and the beginning of a few days exploring Quebec City!

CUMC 2011 was the best undergraduate math experience I've had so far. Aside from meeting other young mathematicians from across the country, CUMC provides a great opportunity to present your research. As a final bonus, I got to explore a city in a completely different part of Canada. In 2012, CUMC will be held at the Okanagan campus of the University of British Columbia in beautiful Kelowna, British Columbia. I am already swearing at Beamer and hope to see you all there!

The Student Committee is accepting applications to fund social and other student events across Canada. The events we have supported in 2012 include the Student Social at the 5th Annual meeting of the Prairie Network for Research in the Mathematical Sciences (University of Regina) and Fields Undergraduate Network Discrete Math meeting (Carleton University). The next application deadline is April 1st. Visit http://studc.math.ca for more information.

Le comité étudiant accepte les demandes de financement d'événements pour les étudiants de partout au Canada. Les événements que nous avons soutenus en 2012 incluent Student Social à la 5ème réunion annuelle du Prairie Network for Research in the Mathematical Sciences (Regina) et la réunion de Fields Undergraduate Network Discrete Math (Carleton). La prochaine date limite pour déposer une demande est le 1er avril. Visitez le http://studc.math.ca pour plus d'informations.

The Distractions Page

Pirates' booty

By Garret Flowers (University of Victoria)

After much swashbuckling, five pirates managed to procure a treasure chest containing 100 gold coins. The pirates, in order of their rank, are Johnny, Orlando, Errol, Jason, and Burt. The decide on the following rules for the distribution of the booty: the highest ranking pirate (the captain) proposes a distribution scheme; if at least half of the pirates agree to it, then the booty is distributed. Otherwise, the pirates commit mutiny, kill the captain, and repeat the process with the new captain. Pirates adhere to a strict moral code:

1. Survival is most important.
2. If survival is guaranteed, then more treasure is preferred.
3. If all else is equal, then pirates prefer to commit mutiny.

The pirates do not trust each other, so no deals can be made. How much gold can Johnny get away with claiming for himself?

If there are 203 pirates, can the captain survive? What about 204?

From Ian Stewart's article "A Puzzle for Pirates".

Johnny can claim 98 gold. Suppose there are only 2 pirates initially: Jason and Burt. Jason can claim all 100 gold, and successfully keep it. If Errol joins, then Errol can give 1 to Burt, and keep 99. Burt agrees with this, since committing mutiny results in no bounty. Thus, if Orlando joins, he can keep 99, while giving 1 coin to Jason. Finally, Johnny can keep 98, and give 1 coin to Errol and 1 to Burt.

Pay close attention to the title of the book

Combinatorial Group Therapy and Topology

CUMC 2011 participants practicing solution to Dr. Johnson's problem

Margin, Volume I, 2011

Image courtesy of CCÉM photosteam on Flickr

Domimatrix

By Christopher Duffy (University of Victoria)

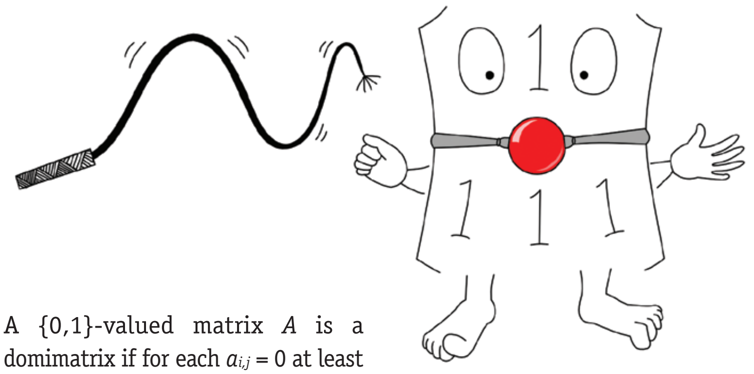


Image courtesy of Jason Siefken (University of Victoria)

A {0,1}-valued matrix A is a domimatrix if for each $a_{i,j} = 0$ at least one of $a_{i+1,j}$, $a_{i-1,j}$, $a_{i,j+1}$ or $a_{i,j-1}$ is equal to 1 (where they exist).

For small values of m and n these matrices are easy to produce. For example, the 3 x 3 matrix with ones on the diagonal is a domimatrix, but the 4 x 4 matrix of the same type is not.

1. Undergraduate:

(a) Construct a 4 x 4 domimatrix with linearly independent columns and 12 zeroes.

(b) Construct a 4 x 4 domimatrix with a zero column and 12 zeroes.

(c) How many zeroes are there in the 1 x n domimatrix with the maximum number of zeroes?

2. Graduate

(a) How many zeroes are there in the 2 x n domimatrix with the maximum number of zeroes?

(b) How many non-equivalent (under reflection and rotation) 3 x 3 domamatrices are there with the maximum number of zeroes?

We notice that that an m x n domimatrix is an m x n grid graph together with a dominating set of that graph. Thus a domimatrix with the maximum number of zeroes corresponds to a minimum dominating set of the m x n grid graph. Graduate: a) $\left\lfloor \frac{n+2}{2} \right\rfloor$, b) 4

Studc in the Community

We've all been told for years that a math degree is good for many things. But when it comes to actually looking for employment (particularly outside of academia), it is sometimes hard to determine exactly what those things are. To address some of the issues, CMS Student Committee hosted a graduate part of the "CMS Studc Fields Trip" event held in conjunction with Fields Undergraduate Network at the CMS Winter Meeting 2011.

The event consisted of a panel discussion and a CV Writing Workshop. The two panelists, Oleksandr Romanko and Professor Hugh C. Williams, discussed how mathematics plays a role in a financial risk management firm and in a highly classified environment, respectively. The CV Writing Workshop consisted of a short presentation, followed by a dynamic discussion and a peer-review of CVs. Discussion included academia and industry-focused CVs, the formatting issues, organization and structure.

We have received very positive feedback regarding both of the events and the slides from both can be found of the Studc web-site at http://studc.math.ca. Watch for future Studc events at the CMS meetings.

David Thomson
Editor

Best food experience: me and four Greek guys at a taverna on a beach in Crete within sight of the Mediterranean. Backgammon and beers for starters, then splitting 2 kilos of rabbit, 1 kilo of goat, salad and potatoes for all.

The CMS Student Committee is looking for proactive mathematics students interested in joining the Committee. Joining Studc is an excellent opportunity to learn the organization of mathematics in Canada and participate in Committee's numerous projects. If you are interested, or know someone who may be, please visit our web-site http://studc.math.ca for more information on Studc projects and to apply. If you have any questions, please contact us at chair-studc@cms.math.ca. The nomination period for these positions will be open until April 15th, 2012. We appreciate early applications.

Le comité étudiant de la SMC est à la recherche d'étudiants en mathématiques dynamiques et intéressés à joindre le comité. Être membre du Studc est une excellente opportunité d'en apprendre plus sur l'organisation des mathématiques au Canada et de participer aux nombreux projets du comité. Si vous êtes intéressés, ou connaissez quelqu'un qui pourrait l'être, veuillez consulter notre site web http://studc.math.ca afin d'obtenir plus d'informations sur les projets du Studc et savoir comment appliquer. Si vous avez des questions, veuillez nous écrire à chair-studc@cms.math.ca. La période des candidatures pour les postes sur le comité sera ouverte jusqu'au 15 avril 2012. Veuillez appliquer le plus tôt possible.

Distraction page's contributors

Garret Flowers

Garret is currently taking a semester off from school. He has temporarily joined a circus and will be touring Asia for the next 3 months.

Christopher Duffy

If it were on the menu, I could be convinced to sample human.

STUDC

CANADIAN MATHEMATICAL SOCIETY
SOCIÉTÉ MATHÉMATIQUE DU CANADA

Notes from the Margin is a semi-annual publication produced by the Canadian Mathematical Society Student Committee (Studc). The Margin strives to publish mathematical content of interest to students, including research articles, profiles, opinions, editorials, letters, announcements, etc. We invite submissions in both English and French. For further information, please visit http://studc.math.ca; otherwise, you can contact the Editor at student-editor@cms.math.ca.

Notes from the Margin est une publication semi-annuelle produite par le comité étudiant de la Société mathématique du Canada (Studc). La revue tend à publier un contenu mathématique intéressant pour les étudiants tels que des articles de recherche, des profiles, des opinions, des éditoriaux, des lettres, des annonces, etc. Nous vous invitons à faire vos soumissions en anglais et en français. Pour de plus amples informations, veuillez visiter http://studc.math.ca; ou encore, vous pouvez contacter le rédacteur en chef à student-editor@cms.math.ca.